1.

(a) FIRST FIND THE MONOPOLIST'S DEMAND.

$$\bar{s} - tx - p = 0 \Rightarrow x = \frac{\bar{s}-p}{t}$$

$$D(p) = \min\left\{\frac{\bar{s}-p}{t}, 1\right\}$$

$$\pi = p\left(\frac{\bar{s}-p}{t}\right)$$

$$d\pi/dp = \frac{\bar{s}-p}{t} - \frac{p}{t} = 0 \Rightarrow p = \bar{s}/2$$

NOW FIND $D(p) = 1 \Rightarrow \dfrac{\bar{s} - \bar{s}/2}{t} = 1$

$$\Rightarrow \bar{s} = 2t$$

SO AS LONG AS $\bar{s} \geq 2t$ A PROFIT MAXIMIZING MONOPOLIST WOULD SELL TO ALL CONSUMERS.

(b)

IN GENERAL THERE IS A DEMAND FUNCTION FOR CONSUMERS TO THE LEFT OF A FIRM LOCATED AT x^* AND A DEMAND FUNCTION FOR CONSUMERS TO THE RIGHT

$$D_L(p) = \begin{cases} \dfrac{\bar{s}-p}{t}, & \text{IF } p \geq \bar{s} - tx^* \\ x^*, & \text{IF } p < \bar{s} - tx^* \end{cases}$$

$$D_R(p) = \begin{cases} \dfrac{\bar{s}-p}{t}, & \text{IF } p \geq \bar{s} - t(1-x^*) \\ (1-x^*), & \text{IF } p < \bar{s} - t(1-x^*) \end{cases}$$

SO THERE ARE TWO KINKS IN THE PROFIT FUNCTION OF THE FIRM.

FIRST CHECK SLOPE OF THE PROFIT FUNCTION WHEN PRICE IS ABOVE BOTH KINK POINTS. THAT IS, WHEN PRICE IS

$$p \geq \max\{\bar{s} - t x^*, \ \bar{s} - t(1-x^*)\}$$

FOR SUCH A PRICE

$$\pi(p) = 2\left[\frac{\bar{s}-p}{t}\right]p$$

$$\pi'(p) = \frac{2}{t}[\bar{s}-2p] \leq \frac{2}{t}\left[\bar{s} - 2\left(\bar{s} - t\min\{x^*, 1-x^*\}\right)\right]$$

$$\leq \frac{2}{t}\left[\bar{s} - 2(\bar{s} - t/2)\right]$$

$$= \frac{2}{t}[t - \bar{s}] < 0 \qquad \text{SINCE } \bar{s} \geq 2t$$

OPTIMUM PRICE IS LESS.

NOW CONSIDER PRICES BETWEEN THE TWO KINK POINTS

$$\max\{\bar{s}-tx^*, \bar{s}-t(1-x^*)\} > p \geq \min\{\bar{s}-tx^*, \bar{s}-t(1-x^*)\}$$

"WITHOUT LOSS OF GENERALITY" ☹ LET $x^* \leq 1/2 \Rightarrow \bar{s}-tx^* > p \geq \bar{s}-t(1-x^*)$

$$\pi(p) = \left[x^* + \frac{\bar{s}-p}{t}\right]p$$

$$\pi'(p) = x^* + \frac{\bar{s}-p}{t} - \frac{p}{t} = x^* + \frac{\bar{s}-2p}{t}$$

$$\leq x^* + \frac{1}{t}\left[\bar{s} - 2\left(\bar{s} - t(1-x^*)\right)\right]$$

$$\leq x^* + \frac{1}{t}\left[t(1-x^*) - \bar{s}\right]$$

$$= \frac{1}{t}\left[t - \bar{s}\right] < 0 \quad \text{SINCE } \bar{s} \geq 2t$$

OPTIMUM IS LESS THAN THE SECOND KINK POINT, SO IN EQ. ALL CUSTOMERS SERVED.

$$p^* = \bar{s} - t \max\{x^*, (1-x^*)\}$$

(c) IT IS EASY TO SHOW THAT WITH ONLY ONE LOCATION THE OPTIMAL PRICE IS $p^* = \bar{s}/2$, DEMAND AT THAT PRICE IS $D(p^*) = 1/2$, AND PROFIT

$$\pi = \bar{s}/4$$

WITH A SECOND LOCATION AT 1, THAT STORES REVENUE WOULD BE THE SAME. THAT IS $\bar{s}/4$. SO A MONOPOLIST WOULD OPEN THE STORE AS LONG AS $f \leq \bar{s}/4$.

THE SOCIAL SURPLUS FROM OPENNING

$$\frac{1}{2}\bar{s} - t\frac{1}{8} - f = \frac{3}{8}\bar{s} - f$$

SOCIAL PLANNER WOULD OPEN THE STORE AS LONG AS $f \leq 3/8\, \bar{s}$

THIS IS AN EXAMPLE OF WHERE A MONOPOLIST WOULD SOMETIME NOT INTRODUCE A NEW LOCATION WHEN A SOCIAL PLANNER WOULD BECAUSE OF NONAPPROPRIABILITY.

(d) NOTE THAT EVEN IF ALL CONSUMERS HAD $t=2$ A MONOPOLIST WOULD WANT TO SELL TO ALL CONSUMERS WHEN $\bar{S}=5$.

TO FIND THE OPTIMAL LOCATION FOR THE MONOPOLIST, WE NEED TO FIND THE LOCATION THAT ALLOWS HER TO CHARGE THE HIGHEST PRICE.

UNLESS THE MONOPOLIST LOCATES CLOSE ENOUGH TO THE LEFT EDGE THE CONSTRAINING CONSUMERS WILL BE LOCATED AT EITHER EDGE. THE OPTIMAL LOCATION WILL BE ONE WHERE THE TRANSPORTATION COSTS ARE EQUAL FOR $x=1$ AND $x=0$. HENCE,

$$2x^* = (1-x^*) \Rightarrow x^* = 1/3$$

THIS IS THE OPTIMAL LOCATION FOR THE MONOPOLIST.

THE SOCIAL PLANNER SEEKS TO MINIMIZE TRANSPORTATION COSTS.

$$C(x^*) = \int_0^{1/4} 2(x^*-x)\,dx + \int_{1/4}^{x^*} (x^*-x)\,dx + \int_{x^*}^{1} (x-x^*)\,dx$$

$$C'(x^*) = \int_0^{1/4} 2\,dx + \int_{1/4}^{x^*} dx - \int_{x^*}^{1} dx = 0$$

$$\Rightarrow \frac{1}{2} + (x^* - 1/4) - (1-x^*) = 0$$

$$\Rightarrow 2x^* - 3/4 = 0 = x^* = 3/8$$

NOTICE THAT FOR A MONOPOLIST WHO WANTS TO SERVE EVERYONE, THE OPTIMAL LOCATION ONLY DEPENDS ON THE RELATIVE t OF THE EDGE CONSUMERS NOT THE 1/4 POINT.

2. THE TRANSPORTATION COST MINIMIZING LOCATION IS CLEARLY AT THE POINT WHERE THE CONSUMERS WITH $t = 2$ LIVE.

FOR THE MONOPOLIST, AS LONG A HE IS LOCATED AT OR AT A NEIGHBORING POINT TO THE HOME OF THE HIGH TRANSPORTATION COST CONSUMERS, THE PRICE CONTRAINT IS THE CONSUMERS LOCATED THE FARTHEST AWAY. HENCE, THOSE THREE LOCATIONS ARE OPTIMAL.

3. IN A COMPETITIVE INSURANCE MARKET PRICE IS EQUAL TO THE EXPECTED LOSS OF THOSE WHO PURCHASE IN EQ. SUPPOSE THE PURCHASING CONSUMERS ARE $B = [\theta^*, \frac{1}{2}]$

COST : $c(\theta^*) = \left(\frac{\frac{1}{2} + \theta^*}{2}\right) L$

FOR EVERYONE IN B TO PURCHASE (AND NOT IN B TO NOT PURCHASE),

$$\alpha \theta^* = \left(\frac{\frac{1}{2} + \theta^*}{2}\right) L$$

$$\Rightarrow \theta^* = \frac{L}{4\alpha - 2L} = \frac{1}{2} \frac{L}{2\alpha - L}$$

$$< \tfrac{1}{2} \quad \text{SINCE } \alpha > L$$